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The analysis of thermal residual stress for Metal Matrix composite with A1/SiC particles.

Hur, Soon Hae

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NAVAL POSTGRADUATE SCHOOL
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THE ANALYSIS OF THERMAL RESIDUAL STRESS FOR
METAL MATRIX COMPOSITE WITH
Al/SiC PARTICLES

by

Hur, Soon Hae
June 1988

Thesis Advisor: Chu Hwa. Lee

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When a metal matrix composite is cooled down to room temperature from the fabrication or annealing temperature, residual stresses are induced in the composite due to the mismatch of the thermal expansion coefficients between the matrix and fiber. A method can be derived for calculating the particles due to differences in thermal expansion coefficients. Special attention is paid to creep deformation in the matrix phase. The analysis shows that considerable internal stresses and creep deformation appear in the composites when subjected to cooling.

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The Analysis of Thermal Residual Stress
for Metal Matrix Composite with Al/SiC Particles

by

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ABSTRACT

When a metal matrix composite is cooled down to room temperature from the fabrication or annealing temperature, residual stresses are induced in the composite due to the mismatch of the thermal expansion coefficients between the matrix and fiber. A method can be derived for calculating the internal stresses appearing in Metal Matrix composites of Al matrix with SiC particles due to differences in thermal expansion coefficients. Special attention is paid to creep deformation in the matrix phase. The analysis shows that considerable internal stresses and creep deformation appear in the composites when subjected to cooling.

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I. INTRODUCTION

A.   METAL MATRIX COMPOSITES(MMCs)

Discontinuously reinforced Metal Matrix Composites(MMCs) represent a group of materials that combine the strength and hardness of the reinforcing phase with the ductility and toughness of the matrix. Aluminum alloys(Al) reinforced with Silicon Carbide(SiC) in particulate, platelet, or whisker form and fabricated by powder metallurgy methods are receiving a great deal of attention from researchers and engineers.

B.   THE ANALYSIS OF MMCs(Al/SICs) IN THE STABLE MEMBER

Previous research has evaluated the dimensional stability, and the thermal and mechanical properties of several Al/SiC MMCs in stable—member applications for missile inertial guidance system. The results reveal that, although the candidate materials from a powder blend of SiC and Al alloy consolidated by Vacuum Hot Pressing(VHP) into cylindrical billets, followed by Hot Isostatic Pressing(HIP) for full densification, have better isotropic properties, these MMCs show some micro creeping behavior in service. The microcreep of these MMCs will affect the dimensional stability of stable members, and is likely to arise from two sources: (1) creep of the metal matrix caused by internal stresses creep conditioned by externally applied stresses and/or (2) phase transformations during the creep condition. A first step toward understanding the cause of the dimensional stability problem is to analyze the influence of internal stresses. The internal stresses are thermal residual stresses when a Metal Matrix Composite is cooled down to room temperature from

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the fabrication or annealing temperature. These residual stresses are induced in the composite due to the mismatch of the thermal expansion coefficients between the aluminum alloy matrix and the silicon carbide particles. The model, based on Eshelby's for mismatch problems with simplified linear elastic material behavior for both particles and matrix, has been used to solve the problem of thermal residual stress [ref 1].

C.   PURPOSE

The purpose of this thesis is to calculate the thermal residual stress of Al/SiC using theoretical methods and then to determine what influence this thermal residual stress has on creep deformation. In order to do this, the thermal residual stresses will be determined by focusing on elastoplastic matrix and elastic inclusions rather than on elastic matrix and elastic inclusions. Therefore, the case where both phases(matrix and inclusions) are perfectly elastic will be treated first. Then, the theoretical results will be compared with previously obtained experimental data for Al/SiC composites.

II. LITERATURE REVIEW

A. MMCs

Metal matrix composites(MMCs), including eutectic composites, are becoming important in applications as structural components which are to be at intermediate and high temperatures. When MMCs are fabricated at high temperature or annealed at certain high temperatures, the MMCs have undesirable properties, such as low tensile yield and ultimate strengths. Those results are mainly due to residual stresses that are caused by the mismatch of the thermal expansion coefficient between the matrix and fiber. The residual stresses so induced have been observed in tungsten fiber/copper composites and in SiC whisker/6061 Al composites and were based on analyzed a one—dimensional model for a continuous fiber system or spherical particle system. The model, based on Eshelby's equivalent inclusion method, has been used to solve the problem of thermal residual stresses[ref 2].

B. ELASTIC MATRIX AND ELASTIC INCLUSIONS

Let's consider a two—phase suspension of finite extent in which one phase is a matrix while the other is in the form of inclusions. It is assumed that the inclusions can be treated as identical ellipsoids with corresponding axes aligned. Further, the inclusions are assumed to be randomly distributed in the matrix in such a way that the suspension is homogeneous on a macroscopic scale. The task of the present section is to solve Eshelby's transformation problem for the case where the fractional volume f of the inclusions is finite. Following Eshelby [ref 2], we consider first an auxiliary problem. what is the resulting stress σ_{ij}^{I} in an inclusion when every inclusion, having isotropic plastic moduli equal to those of the matrix, is subjected

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to a uniform transformation strain ϵ_{ij}^{T} ? To find the exact solution appears to be impossible since no detailed information is given about the spatial distribution of the inclusions. All we may find in this situation are approximate average fields. The average constrained strain ϵ_{ij}^{-C} in any inclusion may be written in the form

$$\epsilon_{ij}^{-C} = \epsilon_{ij}^{C'} + \epsilon_{ij}^{C''} \qquad (1)$$

Here, $\epsilon_{ij}^{C'}$ is the constrained strain in a typical inclusion which would be produced if it alone is embedded in an infinite elastic body. This term is given, according to Eshelby[ref], as

$$\epsilon_{ij}^{C'} = S_{ijkl} \, \epsilon_{ij}^{T} \qquad (2)$$

Where S_{ijkl} is Eshelby's tensor which depends on the aspect ratios of the ellipsoidal inclusion and Poisson's ratio.(Here and in the following we use the usual summation convention: the range of subscripts is form 1 to 3). $\epsilon_{ij}^{-C''}$ is a contribution due to all the surrounding inclusions and the presence of free boundary; this term is entirely elastic, and is assumed to be eiffectively constant in and around the typical inclusion. Then, it will be a good approximation to regard $\epsilon_{ij}^{-C''}$ as the average elastic strain in the matrix for the contribution of any one inclusion to the average field; outside it is negligible when there are a great number of inclusion in the material. For a homogeneous elastic body in static equilibrium, a volume integral of any component of the elastic strain associated with an internal stress should vanish provided that the integration is taken throughout the body. Thus

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$$(1-f)\,\epsilon_{ij}^{-C''} + f(\epsilon_{ij}^{-C} - \epsilon_{ij}^{-T}) = 0 \qquad (3)$$

From (1), (2) and (3), we have

$$\epsilon_{ij}^{-C} = (1-f)S_{ijkl}\,\epsilon_{kl}^{T} + f\epsilon_{ij}^{T} \qquad (4)$$

The average internal stress in any inclusion is given by

$$\sigma_{ij}^{-I} = C_{ijkl}(\,\epsilon_{kl}^{-C} - \epsilon_{kl}^{T}\,) \qquad (5)$$

Where C_{ijkl} is the elastic moduli of the matrix (and also of the inclusions in the present situation) [ref 3]. Where the inclusions have elastic moduli differing from those of the matrix, the same approach as used by Eshelby [ref 2] will be applicable. The essence of this approach is terms of equivalent inclusions. Let C_{ijkl}^{*} be the elastic moduli of the inclusions, and ϵ_{ij}^{T*} be a uniform transformation strain of every inclusion. Then the transformation strain of the equivalent inclusions, ϵ_{ij}^{T}, can be determined by solving a set of equations

$$C_{ijkl}(\,\epsilon_{kl}^{-C} - \epsilon_{kl}^{T}\,) = C_{ijkl}^{*}(\,\epsilon_{kl}^{-C} - \epsilon_{kl}^{T*}\,) \qquad (6)$$

with (4). The average stress in any inclusion is given by (5), of course. Consider a composite specimen which is free of stress at a references temperature T_0. Let the matrix and the inclusions have isotropic linear thermal expansion coefficients a and a^{*}, respectively. The elastic state when the specimen is subjected to a uniform

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temperature change, $\Delta T_0 = T - T_0$, is described by the situation where the matrix and the inclusions undergo uniform transformation strains $a \Delta T \delta_{ij}$ and $a^* \Delta T \delta_{ij}$, respectively. Here δ_{ij} is the Kronecker delta. The resulting state of stress can be analyzed by putting

$$\epsilon^{T*}_{ij} = (a^* - a) \Delta T \, \delta_{ij} \tag{7}$$

The overall thermal expansion coefficients a^{**} can be expressed as a function of a, a^*, C_{ijkl}, C^*_{ijkl}, f and the aspect ratios of the inclusions.

C. ELASTOPLASTIC MATRIX AND ELASTIC INCLUSIONS

When the temperature change is great, one should expect plastic deformation, either in the matrix and/or in the inclusions, which relaxes the internal stress accumulated. An important practical case, the case where the plastic deformation occurs only in the matrix, will be considered in the following. Further, for simplicity, we assume that the plastic strain is uniform throughout the matrix and the plastically nondeforming inclusions can be treated as identical spheroids. Then, the plastic strain in the matrix, ϵ^P_{ij}, may be written in consideration of a symmetric mode of plastic deformation and the volume constant law, as:

$$\epsilon^P_{ij} = \epsilon_p [\delta_{3i} \delta_{3j} - \frac{1}{2}(\delta_{1i} \delta_{1j} + \delta_{2i} \delta_{2j})] \tag{8}$$

Where ϵ_p is a temperature–dependent parameter. In this case, the overall strain ϵ^{**}_{ij} of the composite is given, with use of Equation $\overline{\epsilon}_{ij} = f \epsilon^T_{ij}$, by

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$$\epsilon_{ij}^{**} = a\Delta T \delta_{ij} + \epsilon_{ij}^{P} + f\epsilon_{ij}^{T} \qquad (9)$$

Where ϵ_{ij}^{T} is calculated from (4) and (6) with

$$\epsilon_{ij}^{T*} = (a^{*} - \alpha)\Delta T \delta_{ij} - \epsilon_{ij}^{P} \qquad (10)$$

Now, the temperature dependence of ϵ_p will be found through an energy—balance consideration. Since energy dissipative process is involved in plastic deformation, the decrease in the elastic energy, $-\delta E_{el}$, corresponding to a virtual plastic deformation $\delta\epsilon_p$, should be balanced against the dissipation energy. If the matrix material has a constant flow stress σ_y in a uniaxial deformation, the condition is written as:

$$-\delta E_{el} = (1-f)\sigma_y |\delta\epsilon_p| \quad (\sigma_y > 0) \qquad (11)$$

Where $E_{el} = -\frac{1}{2} f \sigma_{ij}^{-I} \epsilon_{ij}^{T*}$
$$= C_1\epsilon_p^2 + C_2\epsilon_p(a^{*} - \alpha)\Delta T$$
$$+ C_3(a^{*} - \alpha)^2\Delta T^2 \qquad (12)$$

It is noted that when the composite is elastoplatic the following condition should be satisfied:

$$(E_{el})_I > (E_{el})_{II}$$

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Where the subscripts I and II refer to the entirely elastic case and the elastoplastic case, respectively[ref 3].

D. CREEP

If a stress is suddenly applied to pure metals, some solid solutions and most engineering alloys at a temperature near or greater than 0.5Tm(where Tm is the absolute melting point of the metal or alloy) deformation proceeds as shown in Fig. 2.1. The initial application of stress causes an instantaneous elastic strain ϵ_0 to occur. If the stress is sufficiently high an initial plastic deformation ϵ_p also occurs. At low temperatures, significant deformation ceases after the initial application of stress and an increase in stress is required to cause further deformation. At elevated temperatures, deformation under a constant applied load continues with time. The early stage of such deformation called primary creep is characterized by an initially high creep rate $d\epsilon/dt$ which gradually decreases with time. Eventually a linear variation of creep strain accumulation with time is observed. This steady—state creep region is characterized by a constant minimum creep rate. Steady—state creep rates depend significantly on stress and temperature and are used frequently to compare the creep resistance among alloys. After significant deformation in steady—state creep, necking occurs or sufficient internal damage in the form of voids or cavities accumulate to reduce the cross—sectional area resulting in an increase in stress and creep rate. The process accelerates rapidly and failure occurs. This region of the creep curve is called tertiary creep. Fig. 2.1 also shows the derivative of the creep curve or the creep rate curve. Fig. 2.1 represents the most common creep strain/time behavior and Fig. 2.2 illustrates the strain rate/time behavior at temperatures below approximately 0.3Tm. In Fig. 2, only transient creep is

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observed which is characterized by a continuously decreasing creep rate that approaches zero as the inverse of time. Such low—temperature creep behavior is called logarithmic creep. Fig. 2.2 also illustrates the creep rate/time behavior of alloys that exhibit a continuously increasing creep rate in the early stages of creep. Such alloys have been designated Class 1 alloys. Creep by viscous dislocation glide results when dislocations glide or move in slip planes under the action of an applied stress. These dislocations drag along solute atoms attracted to the strain fields of the dislocations. In order for the dislocations to move the solute atmospheres must diffuse in the direction of dislocation motion. At lower temperatures($\sim$0.3Tm) or at higher stress levels, creep occurs by thermally activated dislocation glide. Under these conditions dislocations can overcome barriers to motion without dislocation climb. The localized motion of short segments of small dislocations is important in overcoming barriers. Because of the larger contribution of stress in thermally activated dislocation glide a temperature and stress dependence for creep different from that for dislocation climb or viscous glide will be observed.[ref 6]
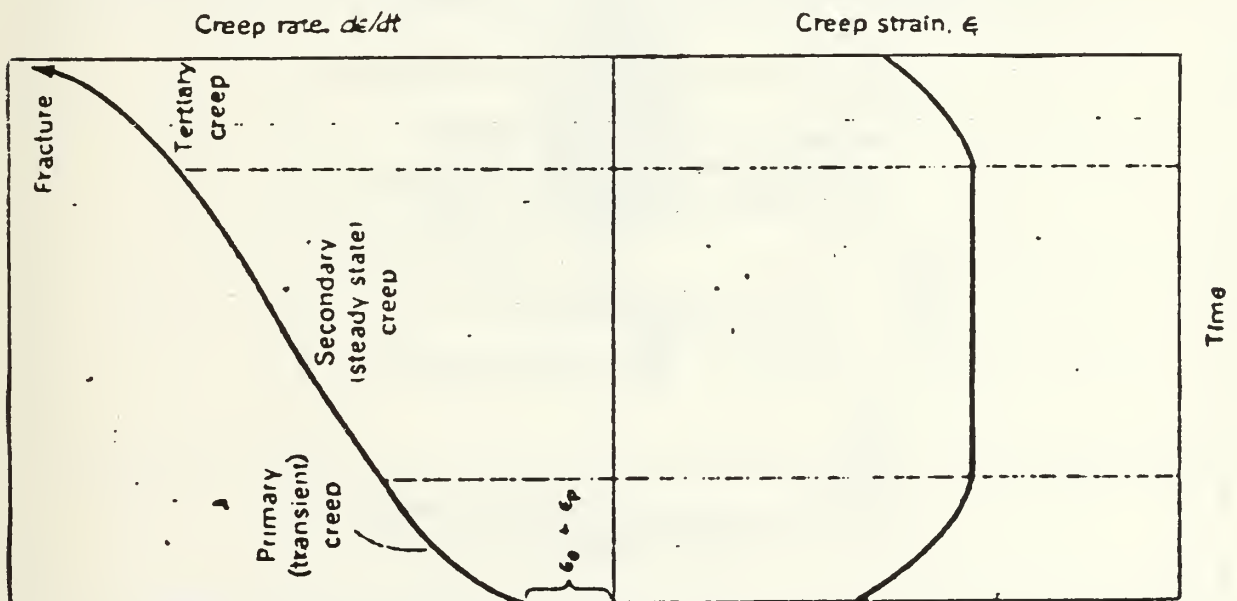
Fig. 2.1 Variation of creep and creep rate with time

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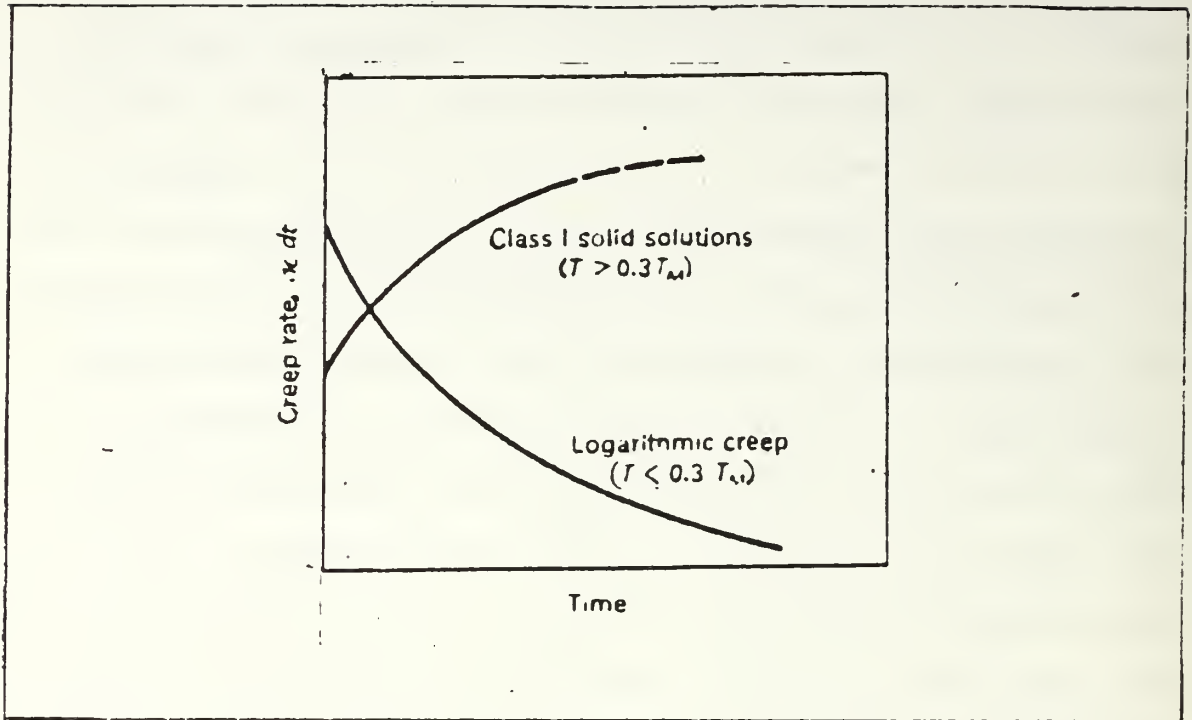
Fig. 2.2 Creep rate vs time
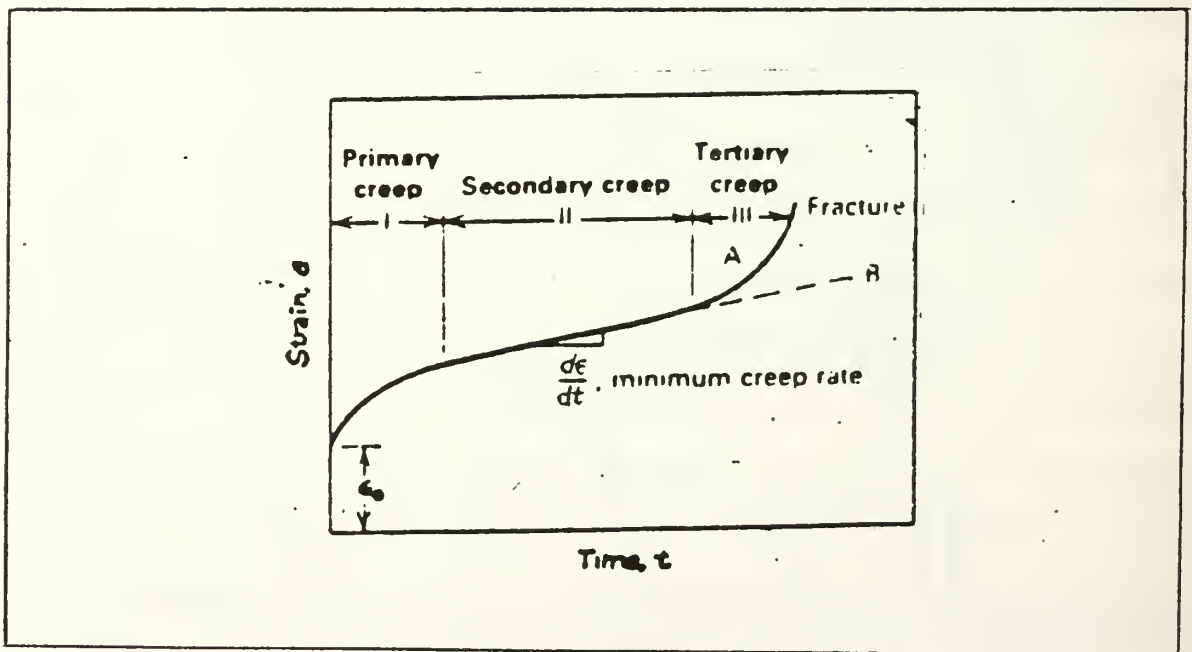
Fig. 2.3 Typical creep curve

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III. THEORETICAL BACKGROUND AND DERIVATION

A.   THEORETICAL BACKGROUND

To simulate Metal Matrix Composites(MMCs), we consider an infinite isotropic body containing an ellipsoidal fiber as shown in Fig. 3.1. The infinite body(composite) experiences a temperature $T1 - T0$, where $T1$ is assumed to be larger than the relaxation temperature of the matrix Tr and of course, $Tr>T0$. To make the problem simpler, the case of one fiber embedded in the infinite body is considered here.
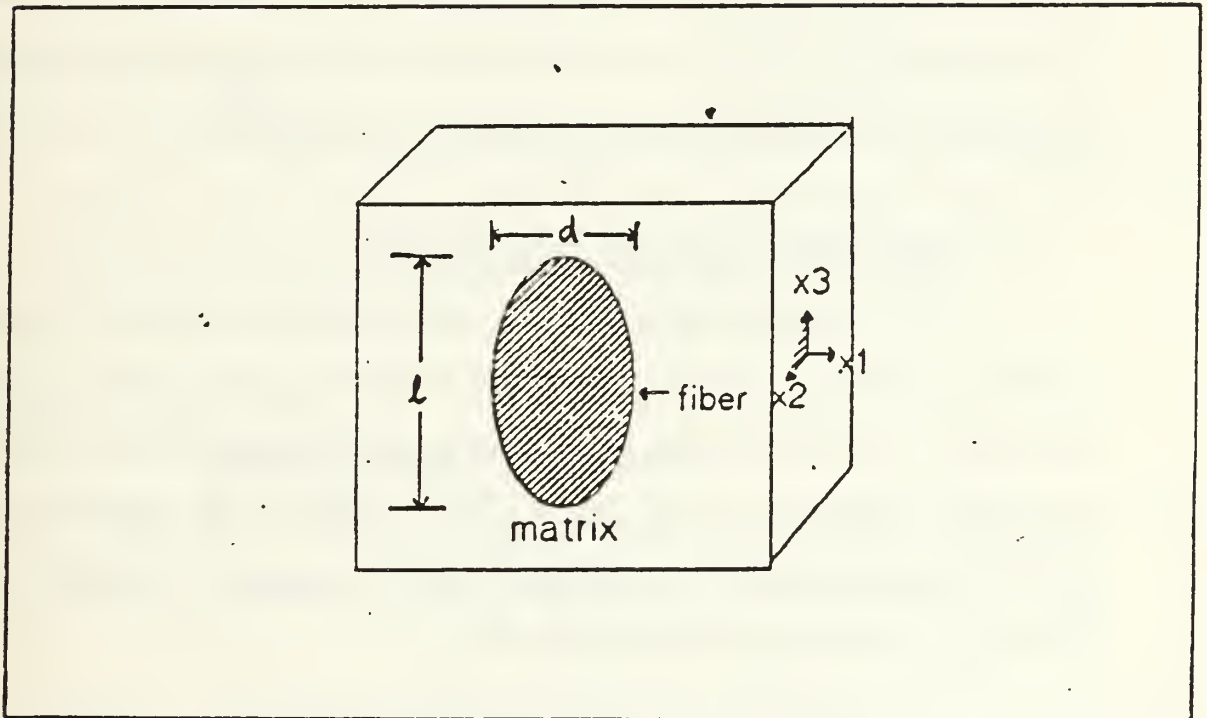
Fig. 3.1. Theoretical model

Based on the model in Fig 2, let us consider the case of elastic inclusions–elastic matrix, elastoplastic matrix–elastic inclusions and creep

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deformation:

1. Elastic Inclusions and Matrix

First let us consider the average internal constrain strain ϵ_{ij}^{-C}. To find ϵ_{ij}^{-C}, we insert Equation(1) and Equation(3) into Equation(4) and find $\epsilon_{ij}^{-C}=(1-f)S_{ijkl}\epsilon_{kl}^{T} + f\epsilon_{ij}^{T}$, which represents the average internal constrain in any inclusion in terms of transformation strain. Next, the average internal stress in any inclusion can be represented by Equation(5). Here, ϵ_{ij}^{T} is an unknown term. Therefore, to find this unknown term we must consider Equation (6) and Equation (4). In Equation (6), * represents the inclusion, and ϵ_{ij}^{T*} represents the uniform transformation strain for every inclusion. Once we have determined ϵ_{ij}^{T}, according to the Equation $\overline{\epsilon}_{ij} = f\epsilon_{ij}^{T}$ we can determine the overall strain of the specimen, and in accordance with Equation(7) we can determine the uniform transformation ϵ_{ij}^{T*}.

2. Elastoplastic Matrix and Elastic Inclusions

The case for elastoplastic matrix—elastic inclusions is different from that of elastic inclusion and matrix. This is due to the fact that in this case plastic deformation occurs only inside the matrix, so we must consider the plastic strain in the matrix. Therefore we can find ϵ_{ij}^{P} in accordance with Equation(8) and Equation(10). From $E_{el} = -\frac{1}{2}f\sigma_{ij}^{-I}\epsilon_{ij}^{T*}$ we can represent $\sigma_{ij}^{-I}=F(\epsilon_{ij}^{P})$ and $\epsilon_{ij}^{T*}=F(\epsilon_{ij}^{P})$. After representing them this way,

$$E_{el} = -\frac{1}{2}f\sigma_{ij}^{-I}\epsilon_{ij}^{T*}$$
$$=C1\,\epsilon_{p}^{2} + C2\,\epsilon_{p}(a^{*}-a)\Delta T + C2\,\epsilon_{p}(a^{*}-a)^{2}\Delta T^{2} \qquad (14).$$

Differentiating this with respect to ϵ_p, we can obtain the value of ϵ_p as shown below:

$$\epsilon_p = -\frac{C2}{2C1}(a^* - a)\Delta T \mp \frac{1}{2C1}(1-f)\sigma_y \qquad (15)$$

Therefore, using the value of ϵ_p to find the value ϵ_{ij}^P, the value of σ_{ij}^{-I} can be obtained, and inserting this value into Equation(16), we are ultimately able to obtain the total average stress in the matrix.[ref 9]

$$< \sigma_{ij}^{-I} >_m = C_{ijkl} < \epsilon_{kl} >_{V_R} = -f\,\sigma_{ij}^{-I} \qquad (16)$$

3. Creep

Applying the value of the internal stress(above) to Equation(13). we can determine creep deformation. Following from this theoretical base, let us first calculate mathematically the thermal residual stress of Al/SiC MMCs.

B. DERIVATION

1. Elastic Inclusions and Elastic Matrix

First let us calculate ϵ_{ij}^T for the case of elastic inclusions and elastic matrix. In order to determine ϵ_{ij}^T, we rewrite Equation(4) in matrix form:

$$
\begin{bmatrix} \epsilon_{11}^{-C} \\ \epsilon_{22}^{-C} \\ \epsilon_{33}^{-C} \\ \epsilon_{44}^{-C} \\ \epsilon_{55}^{-C} \\ \epsilon_{66}^{-C} \end{bmatrix} = (1-f)
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\
S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\
S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\
S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\
S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\
S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}
\end{bmatrix}
\begin{bmatrix} \epsilon_{11}^{f} \\ \epsilon_{22}^{f} \\ \epsilon_{33}^{f} \\ \epsilon_{44}^{f} \\ \epsilon_{55}^{f} \\ \epsilon_{66}^{f} \end{bmatrix} + f
\begin{bmatrix} \epsilon_{11}^{f} \\ \epsilon_{22}^{f} \\ \epsilon_{33}^{f} \\ \epsilon_{44}^{f} \\ \epsilon_{55}^{f} \\ \epsilon_{66}^{f} \end{bmatrix}
$$

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Simplifying this matrix we obtain:

$$
\begin{bmatrix} \bar{\epsilon}_{11}^{C} \\ \bar{\epsilon}_{22}^{C} \\ \bar{\epsilon}_{33}^{C} \\ \bar{\epsilon}_{44}^{C} \\ \bar{\epsilon}_{55}^{C} \\ \bar{\epsilon}_{66}^{C} \end{bmatrix}
= (1-f)
\begin{bmatrix}
\epsilon_{11}^{t} S_{11} f + S_{12}\,\epsilon_{11}^{t} + S_{13}\,\epsilon_{33}^{s} \\
S_{21}\epsilon_{11}^{t} + \epsilon_{22}^{s}(\,S_{22} + f\,) + S_{23}\epsilon_{33}^{s} \\
S_{31}\epsilon_{11}^{t} + S_{32}\,\epsilon_{22}^{s} + \epsilon_{33}^{s}(\,S_{33} + f\,) \\
\epsilon_{44}^{t}(S_{44} + f\,) \\
\epsilon_{55}^{s}(S_{55} + f\,) \\
\epsilon_{66}^{s}(S_{66} + f\,)
\end{bmatrix}
+ f
\begin{bmatrix} \epsilon_{11}^{t} \\ \epsilon_{22}^{s} \\ \epsilon_{33}^{t} \\ \epsilon_{44}^{t} \\ \epsilon_{55}^{s} \\ \epsilon_{66}^{s} \end{bmatrix}
$$

Further, simplifying this matrix we find:

$$\bar{\epsilon}_{11}^{C} = \epsilon_{11}^{t}(1-f)S_{11} + \epsilon_{22}^{s}S_{12}(1-f) + \epsilon_{33}^{s}S_{13}(1-f) + f\epsilon_{11}^{t}$$

$$\bar{\epsilon}_{22}^{C} = \epsilon_{11}^{t}(1-f)S_{21} + \epsilon_{22}^{s}S_{22}(1-f) + \epsilon_{33}^{s}S_{23}(1-f) + f\epsilon_{22}^{s}$$

$$\bar{\epsilon}_{33}^{C} = \epsilon_{11}^{t}(1-f)S_{31} + \epsilon_{22}^{s}S_{32}(1-f) + \epsilon_{33}^{s}S_{33}(1-f) + f\epsilon_{33}^{s}$$

$$\bar{\epsilon}_{44}^{C} = \epsilon_{44}^{s}(1-f)S_{44} + f\epsilon_{44}^{t}$$

$$\bar{\epsilon}_{55}^{C} = \epsilon_{55}^{s}(1-f)S_{55} + f\epsilon_{55}^{s}$$

$$\bar{\epsilon}_{66}^{C} = \epsilon_{66}^{s}(1-f)S_{66} + f\epsilon_{66}^{s} \tag{17}$$

Refer to Appendix (A) for the values of S_{ijkl}.

Next, let us rewrite Equation(6) in matrix form.

$$
G
\begin{bmatrix}
(1-\nu) & \nu & \nu & 0 & 0 & 0 \\
\nu & (1-\nu) & \nu & 0 & 0 & 0 \\
\nu & \nu & (1-\nu) & 0 & 0 & 0 \\
0 & 0 & 0 & 2\mu & 0 & 0 \\
0 & 0 & 0 & 0 & 2\mu & 0 \\
0 & 0 & 0 & 0 & 0 & 2\mu
\end{bmatrix}
\left[
\begin{bmatrix} \bar{\epsilon}_{11}^{C} \\ \bar{\epsilon}_{22}^{C} \\ \bar{\epsilon}_{33}^{C} \\ \bar{\epsilon}_{44}^{C} \\ \bar{\epsilon}_{55}^{C} \\ \bar{\epsilon}_{66}^{C} \end{bmatrix}
-
\begin{bmatrix} \epsilon_{11}^{t} \\ \epsilon_{22}^{s} \\ \epsilon_{33}^{s} \\ \epsilon_{44}^{t} \\ \epsilon_{55}^{s} \\ \epsilon_{66}^{s} \end{bmatrix}
\right]
=
$$

$$
H
\begin{bmatrix}
(1-\nu) & \nu & \nu & 0 & 0 & 0 \\
\nu & (1-\nu) & \nu & 0 & 0 & 0 \\
\nu & \nu & (1-\nu) & 0 & 0 & 0 \\
0 & 0 & 0 & 2\mu & 0 & 0 \\
0 & 0 & 0 & 0 & 2\mu & 0 \\
0 & 0 & 0 & 0 & 0 & 2\mu
\end{bmatrix}
\left[
\begin{bmatrix} \bar{\epsilon}_{11}^{C} \\ \bar{\epsilon}_{22}^{C} \\ \bar{\epsilon}_{33}^{C} \\ \bar{\epsilon}_{44}^{C} \\ \bar{\epsilon}_{55}^{C} \\ \bar{\epsilon}_{66}^{C} \end{bmatrix}
-
\begin{bmatrix} \epsilon_{11}^{t}{}^{*} \\ \epsilon_{22}^{s}{}_{*} \\ \epsilon_{33}^{s}{}_{*} \\ \epsilon_{44}^{t}{}_{*} \\ \epsilon_{55}^{s}{}_{*} \\ \epsilon_{66}^{s} \end{bmatrix}
\right]
$$

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Simplifying the above matrix, we have the following:

$$\epsilon_{11}^{-C}(1-\nu)J+\epsilon_{22}^{-C}\nu J+\epsilon_{33}^{-C}\nu J=(1-\nu)G\epsilon_1{}_1^{\}} + \nu G\epsilon_2{}_2^{\}}+\nu G\epsilon_3{}_3^{\}}-(1-\nu)H\epsilon_1{}_1^{\}*}$$
$$-\nu H\epsilon_2{}_2^{\}*}-\nu H\epsilon_3{}_3^{\}*}$$

$$\epsilon_{11}^{-C}\nu J+\epsilon_{22}^{-C}(1-\nu)J+\epsilon_{33}^{-C}\nu J=\nu G\epsilon_1{}_1^{\}} + (1-\nu)G\epsilon_2{}_2^{\}}+\nu G\epsilon_3{}_3^{\}}-\nu H\epsilon_1{}_1^{\}*}-$$
$$(1-\nu)H\epsilon_2{}_2^{\}*}-\nu H\epsilon_3{}_3^{\}*}$$

$$\epsilon_{11}^{-C}\nu J+\epsilon_{22}^{-C}\nu J+\epsilon_{33}^{-C}(1-\nu)J=\nu G\epsilon_1{}_1^{\}}+ \nu G\epsilon_2{}_2^{\}}+(1-\nu)G\epsilon_3{}_3^{\}}-\nu H\epsilon_1{}_1^{\}*}-$$
$$\nu H\epsilon_2{}_2^{\}*}-(1-\nu)H\epsilon_3{}_3^{\}*}$$

$$\epsilon_{44}^{-C} = 0$$
$$\epsilon_{55}^{-C} = 0$$
$$\epsilon_{66}^{-C} = 0 \tag{18}$$

From Equation (17) and (18), if we simplify and solve for ϵ_{ij}^{T}, we obtain:

$$K\epsilon_1{}_1^{\}}+L\epsilon_2{}_2^{\}}+M\epsilon_3{}_3^{\}}=-(1-\nu)H\epsilon_1{}_1^{\}*}-\nu H\epsilon_2{}_2^{\}*}-\nu H\epsilon_3{}_3^{\}*}$$
$$N\epsilon_1{}_1^{\}}+P\epsilon_2{}_2^{\}}+Q\epsilon_3{}_3^{\}}=-\nu H\epsilon_1{}_1^{\}*}-(1-\nu)H\epsilon_2{}_2^{\}*}-\nu H\epsilon_3{}_3^{\}*}$$
$$R\epsilon_1{}_1^{\}}+S\epsilon_2{}_2^{\}}+T\epsilon_3{}_3^{\}}=-\nu H\epsilon_1{}_1^{\}*}-\nu H\epsilon_2{}_2^{\}*}-(1-\nu)H\epsilon_3{}_3^{\}*}$$
$$\epsilon_4{}_4^{\}} = \epsilon_4{}_4^{\}*}$$
$$\epsilon_5{}_5^{\}} = \epsilon_5{}_5^{\}*}$$
$$\epsilon_6{}_6^{\}} = \epsilon_6{}_6^{\}*} \tag{19}$$

Now, we have 6 unknown values and 6 Equations. Therefore, applying Equation(19) to determine ϵ_{ij}^{T}:

$$\epsilon_1{}_1^{\}}=\frac{-Y'L - X'M - HA'(1+\nu)}{K}$$

15

$$\epsilon_{22}^t = \frac{W - VX'}{U}$$

$$\epsilon_{33}^t = \frac{ZU - XW}{UX - XV}$$

$$\epsilon_{44}^t = \epsilon_{44}^{t*}$$

$$\epsilon_{55}^t = \epsilon_{55}^{t*}$$

$$\epsilon_{66}^t = \epsilon_{66}^{t*} \qquad\qquad (20)$$

Putting the above equations into simplified form:

$$\epsilon_{ij}^T = \begin{bmatrix} \epsilon_{11}^t \\ \epsilon_{22}^t \\ \epsilon_{33}^t \\ \epsilon_{44}^t \\ \epsilon_{55}^t \\ \epsilon_{66}^t \end{bmatrix} = \begin{bmatrix} Z' \\ Y' \\ X \\ \epsilon_{44}^{t*} \\ \epsilon_{55}^{t*} \\ \epsilon_{66}^t \end{bmatrix}$$

Here, refer to Appendix(C) for constants B through Z'. Thus, inserting the value ϵ_{ij}^T into Equation(5), which is the average internal stress in any inclusion, we can determine the internal stress.

$$\begin{bmatrix} \bar{\sigma}_{11I}^I \\ \bar{\sigma}_{22I} \\ \bar{\sigma}_{33I} \\ \bar{\sigma}_{44I} \\ \bar{\sigma}_{55I} \\ \bar{\sigma}_{66} \end{bmatrix} = G \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu \end{bmatrix} \begin{bmatrix} BZ'+CY'+DX'-Z' \\ CZ'+BY'+DX'-Y' \\ FZ'+FY'+EX'-X' \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2. Elastoplastic Matrix and Elastic Inclusions

Up to this point we have found the internal stress in the case of an elastic matrix and elastic inclusions. Now we will consider the case of elastoplastic matrix and elastic inclusions. Here also the method of determining the internal

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and elastic inclusions. Here also the method of determining the internal stress is similar to the case of elastic matrix and inclusion. However in this case, since ϵ_{ij}^P, the plastic strain, is included in the term ϵ_{ij}^{T*}, the next case is more complicated. Therefore, before we consider the relationship between ϵ_{ij}^P and ϵ_{ij}^{T*}, let us first look at ϵ_{ij}^{T*}:

$$\epsilon_{ij}^{T*} = (a^* - a)\Delta T\delta_{ij} - \epsilon_p[\delta_{3i}\delta_{3j} - \frac{1}{2}(\delta_{1i}\delta_{1j} + \delta_{2i}\delta_{2j})] \quad (\delta_{ij}: i=j\rightarrow1, i\neq j\rightarrow0). \text{ Rewriting the}$$

above equation in matrix form we have

$$
\begin{bmatrix} \epsilon_{11}^* \\ \epsilon_{22}^* \\ \epsilon_{33}^* \\ \epsilon_{44}^* \\ \epsilon_{55}^* \\ \epsilon_{66}^* \end{bmatrix} = (\alpha^* - \alpha)\Delta T \begin{bmatrix} \delta_{11} \\ \delta_{22} \\ \delta_{33} \\ \delta_{44} \\ \delta_{55} \\ \delta_{66} \end{bmatrix} - \begin{bmatrix} \epsilon_{11}^P \\ \epsilon_{22}^P \\ \epsilon_{33}^P \\ \epsilon_{44}^P \\ \epsilon_{55}^P \\ \epsilon_{66}^P \end{bmatrix}
\tag{21}
$$

But according to Equation(8), ϵ_{ij}^P can be shown as follows:

$$
\begin{bmatrix} \epsilon_{11}^P \\ \epsilon_{22}^P \\ \epsilon_{33}^P \\ \epsilon_{44}^P \\ \epsilon_{55}^P \\ \epsilon_{66}^P \end{bmatrix} = \epsilon_p \begin{bmatrix} \delta_{31}\delta_{31} - 1/2(\delta_{11}\delta_{11} + \delta_{21}\delta_{21}) \\ \delta_{12}\delta_{12} - 1/2(\delta_{12}\delta_{12} + \delta_{22}\delta_{22}) \\ \delta_{33}\delta_{33} - 1/2(\delta_{33}\delta_{33} + \delta_{23}\delta_{23}) \\ \delta_{34}\delta_{34} - 1/2(\delta_{14}\delta_{14} + \delta_{24}\delta_{24}) \\ \delta_{35}\delta_{35} - 1/2(\delta_{15}\delta_{15} + \delta_{25}\delta_{25}) \\ \delta_{36}\delta_{36} - 1/2(\delta_{16}\delta_{16} + \delta_{26}\delta_{26}) \end{bmatrix}
$$

From the above equation we can obtain the following:

$$
\epsilon_{ij}^P = \begin{bmatrix} -1/2\epsilon_p \\ -1/2\epsilon_p \\ \epsilon_p \\ 0 \\ 0 \\ 0 \end{bmatrix}
\tag{22}
$$

Therefore, inserting Equation(22) into Equation(21) and simplifying we obtain:

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$$\epsilon_{11}^{T*} = (a^* - \alpha)\Delta T + \epsilon_p/2$$

$$\epsilon_{22}^{T*} = (a^* - \alpha)\Delta T + \epsilon_p/2$$

$$\epsilon_{33}^{T*} = (a^* - \alpha)\Delta T - \epsilon_p$$

$$\epsilon_{44}^{T*} = 0$$

$$\epsilon_{55}^{T*} = 0$$

$$\epsilon_{66}^{T*} = 0 \tag{23}$$

Next, using Equation(4) and Equation(6), if ϵ_{ij}^{T} is expressed as ϵ_{11}^{-c}, the result is the same as Equation(17). Inserting Equation(17) into Equation(6) and expressing ϵ_{ij}^{T} as ϵ_{ij}^{T*} we obtain:

$$\epsilon_{11}^{T} = Q_1\epsilon_{11}^{T*} - Q_2\epsilon_{22}^{T*} - Q_3\epsilon_{33}^{T*}$$

$$\epsilon_{22}^{T} = P_4\epsilon_{11}^{T*} + P_5\epsilon_{22}^{T*} + P_6\epsilon_{33}^{T*}$$

$$\epsilon_{33}^{T} = P_1\epsilon_{11}^{T*} + P_2\epsilon_{22}^{T*} + P_3\epsilon_{33}^{T*}$$

$$\epsilon_{44}^{T} = 0$$

$$\epsilon_{55}^{T} = 0$$

$$\epsilon_{66}^{T} = 0 \tag{24}$$

Next, in order to find ϵ_{11}^{-c}, we insert the value of ϵ_{ij}^{T} into Equation(14) and simplify:

$$\epsilon_{11}^{-c} = R_1\epsilon_{11}^{T*} + R_2\epsilon_{22}^{T*} + R_3\epsilon_{33}^{T*}$$

$$\epsilon_{22}^{-c} = R_4\epsilon_{11}^{T*} + R_5\epsilon_{22}^{T*} + R_6\epsilon_{33}^{T*}$$

$$\epsilon_{33}^{-c} = R_7\epsilon_{11}^{T*} + R_8\epsilon_{22}^{T*} + R_9\epsilon_{33}^{T*}$$

$$\epsilon_{44}^{-c} = 0$$

$$\epsilon_{55}^{-c} = 0$$

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$$\epsilon_{66}^{-C}=0 \tag{25}$$

Therefore σ_{ij}^{-I} can be found in accordance with Equation(5). (The value of this σ_{ij}^{-I} is different from the value of σ_{ij}^{-I} for elastic inclusion and elastic matrix, the reason being that σ_{ij}^{-I} for elastic inclusion and elastic matrix does not contain ϵ_{ij}^P). Here, in order to determine the value of σ_{ij}^{-I} we must first find the value of ϵ_p. From Equation(10), if we differentiate E_{el} with respect to ϵ_p we can obtain Equation(11). Therefore, from $\delta E_{el} = [\, 2C_1\epsilon_p + C_2(a^* - \alpha)\Delta T]\delta\epsilon_p$ and $-\delta E_{el} = (1-f)\sigma_y \,|\delta\epsilon_p|$, we can obtain the value of ϵ_p. Again, inserting this value of ϵ_p into Equation(22) we can find the value of ϵ_{ij}^P. Finally, by inserting the value of ϵ_{ij}^P into equation(27) we obtain the value of σ_{ij}^{-I}. Utilizing Equation(10) again to find the value of ϵ_p

$$E_{el}= -\frac{1}{2}f \begin{bmatrix} \sigma_{11}^{-I} \\ \sigma_{22}^{-I} \\ \sigma_{33}^{-I} \\ \sigma_{44}^{-I} \\ \sigma_{55}^{-I} \\ \sigma_{66}^{-} \end{bmatrix} \begin{bmatrix} \epsilon_1^* \\ \epsilon_2^* \\ \epsilon_3^* \\ \epsilon_4^* \\ \epsilon_5^* \\ \epsilon_6^* \end{bmatrix} = -\frac{1}{2}f \begin{bmatrix} X_1A' + X_2\epsilon_p \\ X_3A' + X_4\epsilon_p \\ X_5A' + X_6\epsilon_p \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} A' + \epsilon_p/2 \\ A' + \epsilon_p/2 \\ A' - \epsilon_p \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= -\frac{f}{2}\left[\epsilon_p^2\left(\frac{X_2}{2}+\frac{X_4}{2} - X_6\right) + \epsilon_p A'\left(\frac{X_1}{2}+X_2+\frac{X_3}{2}+X_4-X_5+X_6\right)\right.$$
$$\left. + A'^2(\, X_1+X_3+X_5\,)\right] \tag{26}$$

Therefore, differentiating E_{el} in respect to ϵ_p:

$$\delta E_{el} =-f(\, C_2A'_p+ 2\epsilon_pC_1\,)\delta\epsilon_p \tag{27}$$

And inserting Equation(27) into Equation(11) and simplifying obtains:

$$f(\, C_2A'_p+ 2\epsilon_pC_1\,)\delta\epsilon_p=(1-f)\sigma_y \tag{28}$$

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If we find the value of ϵ_p from Equation(28).

$$\epsilon_p = -\frac{C_2}{2C_1}(a^* - \alpha)\Delta T \mp \frac{1}{2C_1}(1-f)\sigma_y \qquad (27)$$

$$(+: \delta\epsilon_p < 0 \rightarrow \text{Cooling}, \; -: \delta\epsilon_p > 0 \rightarrow \text{Heating})$$

If this value of ϵ_p is inserted into Equation(24) and (25), the value of ϵ_{ij}^T and the value of ϵ_{ij}^{-C} can be found, and if these values are inserted into Equation(5) the value of σ_{ij}^{-I} can be determined. Therefore, inserting this value into Equation(16) we are ultimately able to obtain total average stress in the matrix. Refer to Appendix(C) for the constants and actual values.

3. Creep

Up to this point we have been determining the internal stress both in the case of elastic matrix and elastic inclusion and in the case of elastoplastic matrix and elastic inclusion. But in these two cases the actual effect on creep deformation is the internal stress in the case of elastoplastic matrix and elastic inclusion. Accordingly, by inserting the value of the internal stress into Equation (13), we can plot the microcreep deformation phenomenon as a function of time as shown in Fig. 3.2. Refer to Chapter IV part A for more detailed information.
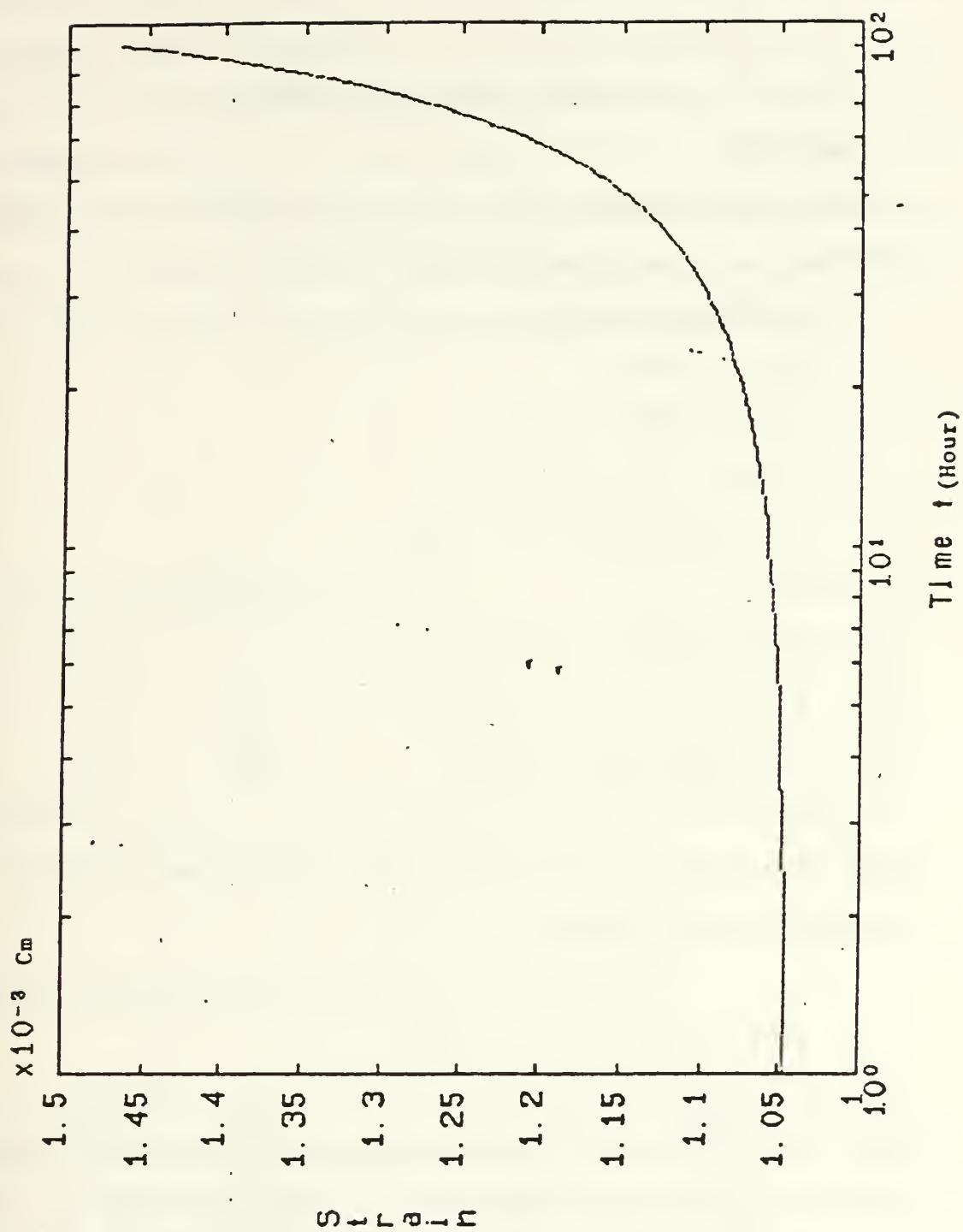
FIG. 3.2 MicroCreep as a function of time t

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IV. RESULTS AND CONCLUSIONS

A.   RESULTS

The thermo—mechanical data of the matrix and fiber for the theoretical calculations are obrtained from the [ref 13].

Annealed 2024 Al matrix:

$$Em = 47.5 \text{ GPa}$$

$$\sigma_y = 47.5 \text{ MPa}$$

$$\nu = 0.3$$

$$a = 23.6 \times 10^{-6} / \text{ K} \qquad (31)$$

SiC fiber:

$$Ei = 427 \text{ GPa}$$

$$f = 0.2$$

$$\alpha^* = 4.3 \times 10^{-6} / \text{ K}$$

$$l/d = 1.5 \qquad (32)$$

Where the average value of the fiber aspect ratio(l/d) was used [ref 13]. The temperature drop ΔT is define as

$$\Delta T = T1 - T_0 \qquad (33)$$

Where T1 is taken as the temperature below which dislocation generation is minimal during the cooling process and T_0 is the room temperature. Thus, for the present composite system ΔT is set equal to $-200K$. From the data given by Equation (31), (32) and the use of $< \sigma_{ij}^{-I} >_m = -f\sigma_{ij}^{-I}$, we have computed the stresses. Next, the thermal residual stresses, averaged in the matrix of SiC fiber/2024 Al, are

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predicted by (17) and the result on $< \sigma_{33}^{-1} >_m$ as plotted in Fig.6, where 33 denote the component along the longitudinal direction(z). The average theoretical thermal residual stress is predicted to be tensile in nature, and the average residual stress in the longitudinal direction to be larger than the average residual stress in the transverse direction [Table 5]. The fiber aspect ratio($\alpha=l/d$) of SiC fibers has been observed to be variable [Appendix A,B]. In the present model we have used the value of l/d, 1.5 to predict the thermal residual stress of the composite.

TABLE 1: The value of ϵ_{ij}^{P} ($\alpha=1,1.5$)

$\alpha(l/d)$	ϵ_{11}^{P}	ϵ_{22}^{P}	ϵ_{33}^{P}
1	−0.0017	−0.0017	−0.0017
1.5	0.0014	0.0014	−0.0028

TABLE 2: The value of ϵ_{ij}^{T*} ($\alpha=1, 1.5$)

$\alpha(l/d)$	ϵ_{11}^{T*}	ϵ_{22}^{T*}	ϵ_{33}^{T*}
1	0.0056	0.0056	0.0056
1.5	0.0025	0.0025	0.0066

TABLE 3: The value of ϵ_{ij}^{-c} (α=1, 1.5)

$\alpha(l/d)$	ϵ_{11}^{-c}	ϵ_{22}^{-c}	ϵ_{33}^{-c}
1	0.0053	0.0053	0.0053
1.5	0.0025	0.0025	0.0060

TABLE 4: The value of ϵ_{ij}^{T} (α=1, 1.5)

$\alpha(l/d)$	ϵ_{11}^{T}	ϵ_{22}^{T}	ϵ_{33}^{T}
1	0.0076	0.0076	0.0076
1.5	0.0025	0.0025	0.0115

TABLE 5: The value of σ_{ij}^{-I} (α=1, 1.5)

$\alpha(l/d)$	σ_{11}^{-I}(MPa)	σ_{22}^{-I}(MPa)	σ_{33}^{-I}(MPa)
1	−276.23	−276.23	−276.23
1.5	−154.82	−154.82	−154.82

TABLE 6: The value of $<\sigma_{ij}^{-I}>_m$ (α=1, 1.5)

$\alpha(l/d)$	$<\sigma_{11}^{-I}>$(MPa)	$<\sigma_{22}^{-I}>$(MPa)	$<\sigma_{33}^{-I}>$(MPa)
1	55.245	55.245	55.245
1.5	30.905	30.905	70.998

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TABLE 7: Comparison value of $< \sigma_{ij}^{-I} >_m$ with TODAY'S

$a(l/d)$	OUR MODELS'S	TAYA'S
	$< \sigma_{33}^{-I} >_m(MPa)$	$< \sigma_{33}^{-I} >_m(MPa)$
1.8	71.156	67.894

As seen in tables 1, 2, 3, 4, 5, 6, and 7 we can observe that as α increases, the values of ϵ_{ij}^P, ϵ_{ij}^{T*}, ϵ_{ij}^{-C}, ϵ_{ij}^T, σ_{ij}^{-I}, and $< \sigma_{ij}^{-I} >_m$ also increase.

Also in Fig. 4.1 we observe that when the aspect ratio(α) increases, the value of the internal stress also increases with it.

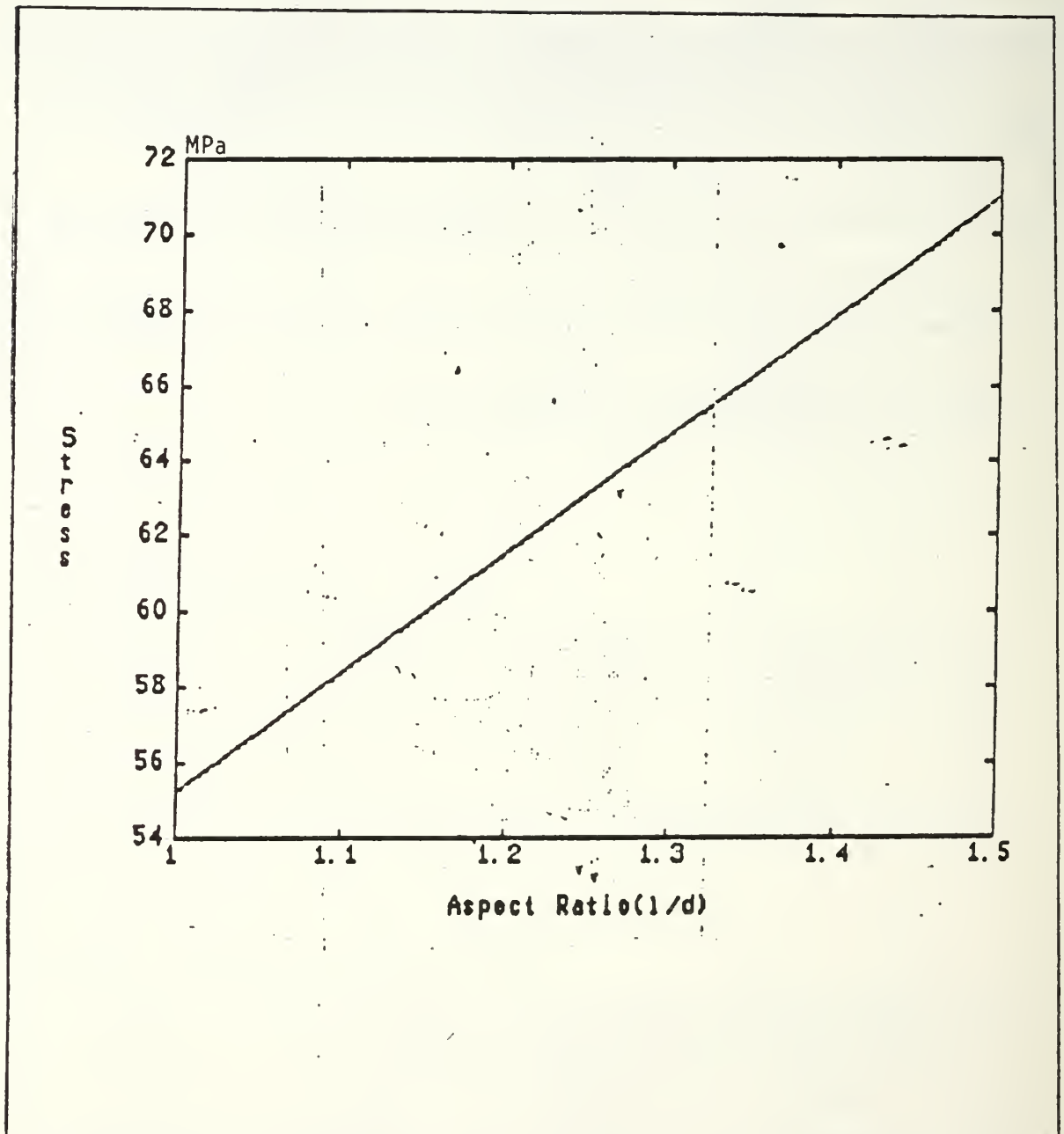Fig. 4.1 Internal stress vs aspect ratio

And in Fig. 4.2 we can also see that the value of the internal stress increases with an increase in temperature.
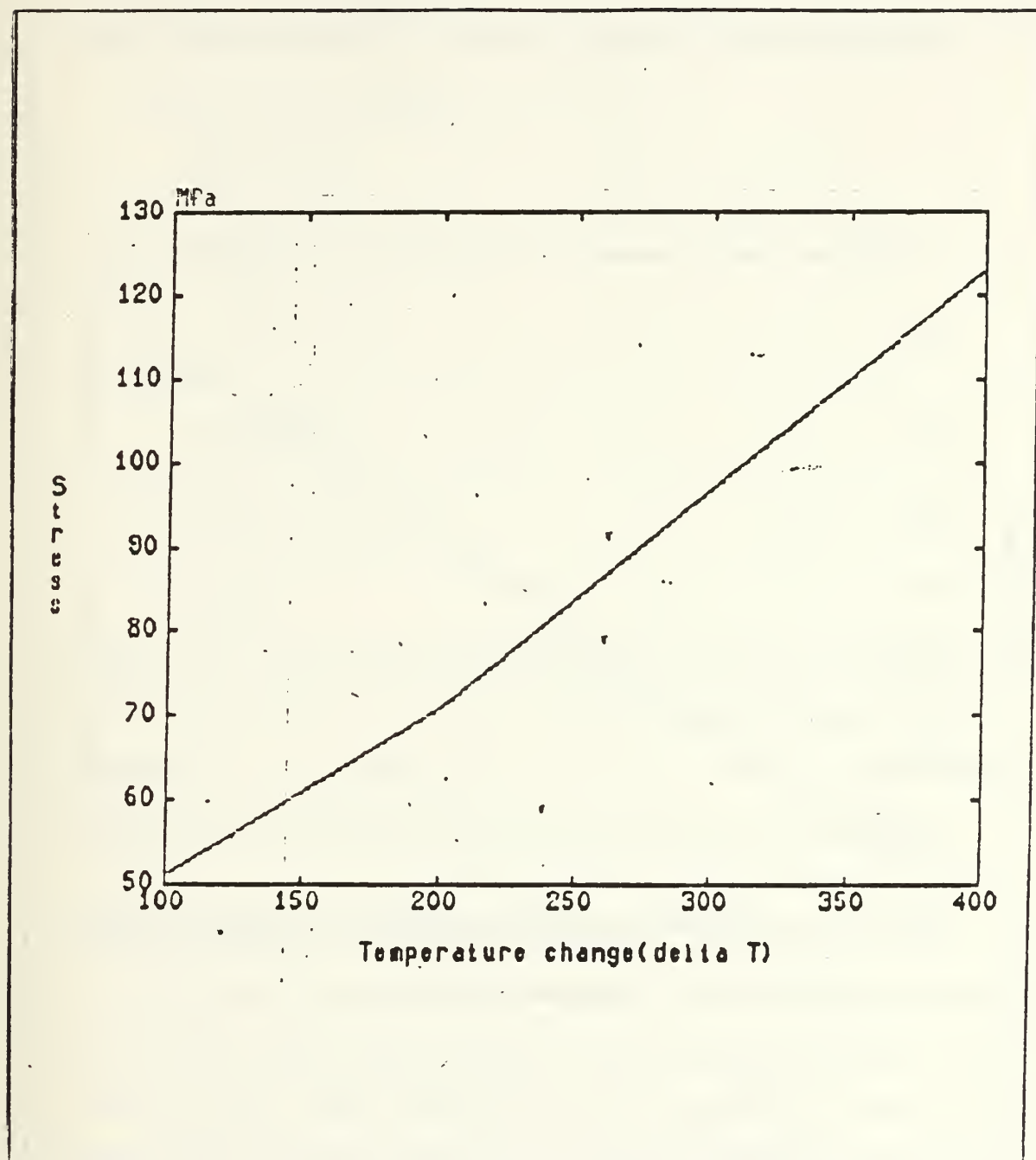

Fig. 4.2 Internal stress vs temperature change(ΔT)

In Fig. 4.3, in the case of an ellipsoid (α=1.5), the values of internal stresses increase with an increase in volume fraction,
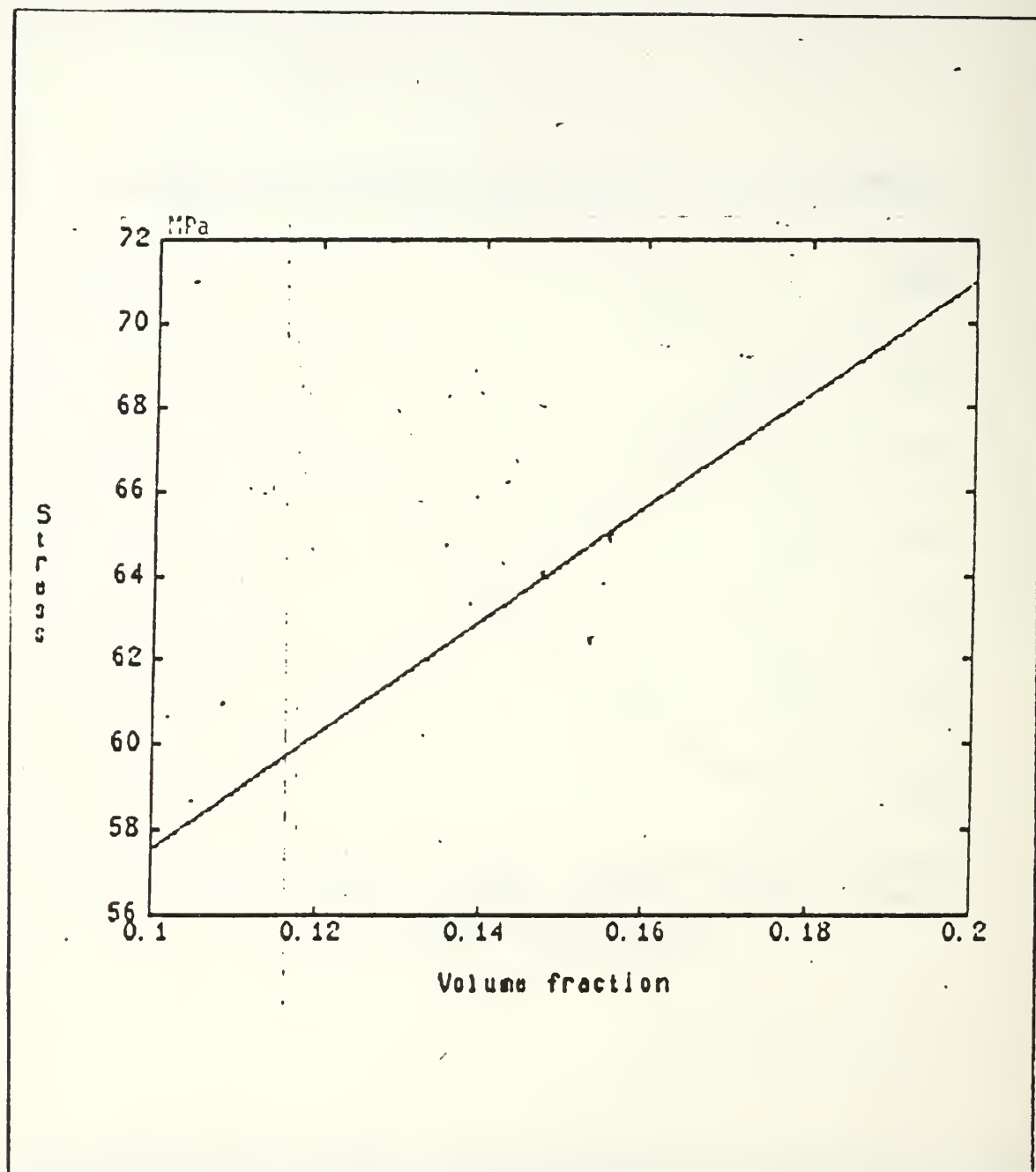
Fig. 4.3 Internal stress vs volume fraction(f)

Next, if we compare the theoretically predicted value of the internal stress with the value determined by R. J. ARSENAULT and M. TAYA, we see that our model's value is larger than the value which is obtained by R. J. ARSENAULT and M. TAYA, as shown in Table 7. They used the material properties as follows:

Annealed 6061 Al matrix:

E=47.5 GPa

σ_y=47.5 MPa

ν =0.33

α =23.6×10^{-6}/K

SiC whisker:

E_i=427GPa

f =0.17

α^*=4.3×10^{-6}/K

l/d=1.8

Finally, Fig. 3.2 is a graph showing the strain as a function of time, when the aspect ratio is 1.5 and the value of the thermal residual stress is inserted into Equation(13). If we analyze Fig. 2.3 we can see that the creep deformation phenomenon is due to the internal thermal residual stress of the MMCs.

B.   CONCLUSIONS AND RECOMMENDATIONS

The object of this research is to obtain the value of the thermal residual stress of an Al/SiC composite using Eshelby's theoretical model, and then to determine what effect this thermal residual stress value produces on creep deformation. Because exact creeping behavior of this material is difficult to determine when analyzing creep deformation, Andrade's model was used with properly selected

constant values. But through this theoretical approach the following conclusions can be obtained.

1. Thermal residual stress due to the difference in thermal expansion coefficients may be estimated.

   a. By means of the theoretical model, we can see that the thermal residual stresses increase when the volume fraction of the inclusions increases.

   b. In the case of sphere inclusions, that is, when the aspect ratio $\alpha=1$, the lateral stress is equal to the longitudinal stress.

   c. In the case of an ellipsoid inclusions where $\alpha=1.5$ the longitudinal stress is greater than the lateral stress.

   d. The thermal residual stresses of Al/SiC composites increased when the value of the aspect ratio of the inclusions increases.

2. Microcreep deformation can be estimated in the model by using the thermal residual stresses and the Andrade's model of creep deformation

   a. Microcreep deformation is due to the internal thermal residual stress of the MMCs.

   b. Dimensional stability of components will be influenced by the behavior of the composite.

3 Recommendations

   a. Presently, this thesis has only dealt with average thermal residual stress from a overall point of view. and we need more detailed local residual stresses surrounding SiC particles should be estimated for analyzing creeping behavior.

   b. In the future, research should be done concerning relaxation during cooling.

   c. The creep behavior used in the current model is the Andrade's approximation. The real creep deformation for this Al/SiC composite should be further studied in order to get better understanding of the microcreep of the material based on approximation values.

APPENDIX A

PROGRAM FOR VALUE OF $S_{ijkl}(\alpha=1.5-5)$

1. PROGRAM(FORTAN)

```
      REAL*8 NU,ALP,G,TMP,S1111,S3333,S1122,S1133,S3311
      NU=0.3
      WRITE(6,90)
      WRITE(6,95)
      ALP=1.5
300   TMP=ALP**2-1.
      G=(ALP*(ALP*DSQRT(TMP)-DLOG(ALP+DSQRT(TMP))))/(TMP**(3./2.))
      S1111=(3.*ALP**2)/(8.*(1.-NU)*TMP)
     ++(1.-2.*NU-9./(4.*TMP))*G/(4.*(1-NU))
      S3333=((1.-2.*NU+(3.*ALP**2-1.)/TMP-(1.-2.*NU+3.*ALP**2/TMP)
     +*G))/(2.*(1.-NU))
      S1122=(ALP**2/(2.*TMP)-(1.-2*NU)-3*G/(4.*TMP))/(4.*(1.-NU))
      S1133=-ALP**2/(2.*(1.-NU)*TMP)+(3.*ALP**2/TMP-(1.-2.*NU))*G
     +/(4.*(1.-NU))
      S3311=-(1.-2.*NU+1./(TMP))/(2.*(1.-NU))+(1.-2.*NU+3./(2.*TMP))
     +*G/(2.*(1.-NU))
      WRITE(6,100) ALP,S1111,S3333,S1122,S1133,S3311
      ALP=ALP+.1
      IF(ALP.GT.5.) GOTO 400
      GOTO 300
90    FORMAT(1X,'      ALP       S1111      S3333     S1122      S1133
     + S3311')
95    FORMAT(1X,'  ===============================================
     +=====')
100   FORMAT(1X,6(F10.5))
400   STOP
      END
```

2. S_{ijkl} VALUE WITH ASPECT RATIO

ALP	S1111	S3333	S1122	S1133	S3311
1.50000	.58078	.39378	.01421	.08396	.01945
1.60000	.58828	.37371	.01603	.08992	.01623
1.70000	.59495	.35518	.01764	.09549	.01350
1.80000	.60089	.33804	.01906	.10071	.01117
1.90000	.60620	.32215	.02032	.10560	.00918
2.00000	.61098	.30739	.02145	.11018	.00747
2.10000	.61529	.29366	.02246	.11448	.00600
2.20000	.61920	.28087	.02337	.11851	.00472
2.30000	.62274	.26893	.02419	.12231	.00362
2.40000	.62598	.25777	.02493	.12588	.00266
2.50000	.62893	.24732	.02560	.12924	.00183
2.60000	.63164	.23752	.02622	.13241	.00111
2.70000	.63412	.22832	.02678	.13540	.00048
2.80000	.63641	.21967	.02729	.13822	-.00007
2.90000	.63852	.21152	.02776	.14090	-.00056
3.00000	.64048	.20384	.02819	.14342	-.00098
3.10000	.64229	.19659	.02859	.14582	-.00134
3.20000	.64397	.18974	.02896	.14809	-.00167
3.30000	.64554	.18326	.02930	.15025	-.00195
3.40000	.64700	.17713	.02962	.15230	-.00219
3.50000	.64836	.17131	.02991	.15425	-.00241
3.60000	.64963	.16579	.03019	.15610	-.00259
3.70000	.65082	.16054	.03044	.15787	-.00276
3.80000	.65194	.15555	.03068	.15955	-.00290
3.90000	.65299	.15081	.03090	.16115	-.00302
4.00000	.65398	.14629	.03111	.16269	-.00312
4.10000	.65491	.14197	.03130	.16415	-.00321
4.20000	.65579	.13786	.03149	.16555	-.00329
4.30000	.65662	.13393	.03166	.16689	-.00335
4.40000	.65740	.13018	.03182	.16818	-.00340
4.50000	.65814	.12659	.03197	.16940	-.00345
4.60000	.65884	.12316	.03212	.17058	-.00348
4.70000	.65951	.11987	.03225	.17171	-.00351
4.80000	.66014	.11671	.03238	.17280	-.00353
4.90000	.66074	.11369	.03250	.17384	-.00355

APPENDIX B

PROGRAM FOR VALUE OF $S_{ijkl}(\alpha=1.0, \infty)$

1. PROGRAM(MATLAB)

```
%
%SPHERE (ALPHA=1.0)
%
for i=1:6;
nu=i/10.;
den=15.*(1-nu);
s11(i)=(7-5*nu)/den;
s12(i)=(5*nu-1)/den;
s13(i)=s12(i);
s31(i)=s12(i);
s33(i)=s11(i);
%
%ALPHA=INFINITY
%
den2=8*(1-nu);
ss11(i)=(5-4.*nu)/den2;
ss12(i)=-(1-4*nu)/den2;
ss13(i)=nu/(2.*(1-nu));
ss31(i)=0;
ss33(i)=0;
end;
result1=[s11;s12;s13;s31;s33;]'
result2=[ss11;ss12;ss13;ss31;ss33]'
```

2. S_{ijkl} VALUE

nu \ S	S11	S12	S13	S31	S33
result1 =					
0.1	0.4815	-0.0370	-0.0370	-0.0370	0.4815
0.2	0.5000	0	0	0	0.5000
0.3	0.5238	0.0476	0.0476	0.0476	0.5238
0.4	0.5556	0.1111	0.1111	0.1111	0.5556
0.5	0.6000	0.2000	0.2000	0.2000	0.6000
0.9	0.6667	0.3333	0.3333	0.3333	0.6667
result2 =					
0.1	0.6389	-0.0833	0.0556	0	0
0.2	0.6563	-0.0312	0.1250	0	0
0.3	0.6786	0.0357	0.2143	0	0
0.4	0.7083	0.1250	0.3333	0	0
0.5	0.7500	0.2500	0.5000	0	0
0.6	0.8126	0.4375	0.7500	0	0

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APPENDIX C

PROGRAM FOR VALUE OF THERMAL RESIDUAL STRESS,

$$\epsilon_{ij}^{P}, \ \epsilon_{ij}^{T*}, \ \epsilon_{ij}^{-C}, \sigma_{ij}^{-I}, \ <\sigma_{ij}^{-I}>_{m}$$

1. PROGRAM(MATLAB)

```
.r:q
%This program computes stress values and constants
%
%           Initial values
%
nu=0.3;
em=47.5e9;
ei=427.0e9;
sy=47.5e6;
f=0.2;
dt=-200
ap=23.6e-6;
as=4.3e-6;
mu=0.;
%
%        Eshelby's tensor components
%
s11=0.6786;
s33=0.0000;
s12=0.0357;
s13=0.2143;
s31=0.0000;
%
%        Constants
%
app=(as-ap)*dt;
g=em/((1.+nu)*(1.-2.*nu));
h=ei/((1.+nu)*(1.-2.*nu));
j=(em-ei)/((1.+nu)*(1.-2.*nu));
a=f+s11*(1.-f);
b=s12*(1.-f);
c=s13*(1.-f);
e=f+s33*(1.-f);
d=s31*(1.-f);
```

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continued

```
t1=r1+r2+r3-q1+q2+q3;
t2=(r1+r2-2*r3)/2.-(q1-q2+2*q3)/2.;
t3=r4+r5+r6-p4-p5-p6;
t4=(r4+r5-2*r6)/2.-(p4+p5-2*p6)/2.;
t5=r7+r8+r9-p1-p2-p3;
t6=(r7+r8-2*r9)/2.-(p1+p2-2*p3)/2.;
x1=g*((1.-nu)*t1+nu*(t3+t5));
x2=g*((1.-nu)*t2+nu*(t4+t6));
x3=g*((1.-nu)*t3+nu*(t1+t5));
x4=g*((1.-nu)*t4+nu*(t2+t6));
x5=g*((1.-nu)*t5+nu*(t1+t3));
x6=g*((1.-nu)*t6+nu*(t2+t4));
c1=((x2+x4)/2.-x6)*(f/2.);
c2=((x1+x3)/2.+x2+x4-x5+x6)*(f/2.);
%
%          Temprature dependent parameter
%
ep=-c2*app/(2.*c1)+(1.-f)*sy/(2.*c1);
%
%          Elastic moduli of the matricies
%
aaa=g*[(1-nu) nu nu 0 0 0;nu (1-nu) nu 0 0 0;nu nu (1-nu) 0 0 0;
       0 0 0 2.*mu 0 0;0 0 0 0 2.*mu 0;0 0 0 0 0 2.*mu]';
%
%          Plastic strain in the matrix
%
eijp=[-ep/2 -ep/2 ep 0 0 0]'
%
%          Uniform transformation strain of inclusion
%
eijts=[app+ep/2 app+ep/2 app-ep 0 0 0]'
%
```

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```
%
eijt=[q1*(app+ep/2)-q2*(app+ep/2)-q3*(app-ep) ..
      p4*(app+ep/2)+p5*(app+ep/2)+p6*(app-ep) ..
      p1*(app+ep/2)+p2*(app+ep/2)+p3*(app-ep) 0 0 0]
eij_c=[r1*(app+ep/2)+r2*(app+ep/2)+r3*(app-ep) ..
      r4*(app+ep/2)+r5*(app+ep/2)+r6*(app-ep) ..
      r7*(app+ep/2)+r8*(app+ep/2)+r9*(app-ep) 0 0 0]
%
%         Average internal stress in inclusion
%
sij_i=aaa*(eij_c-eijt)
%
%         Average stress in the matrix
%
sijm=-f*sij_i
```

2. CONSTANTS OF ELASTIC MATRIX AND ELASTIC INCLUSIONS

$B = S_{11}(1-f) + f$

$C = S_{12}(1-f) + f$

$D = S_{13}(1-f)$

$E = S_{33}(1-f) + f$

$F = S_{31}(1-f)$

$G = E_m/(1+\nu)(1-2\nu)$

$H = E_i/(1+\nu)(1-2\nu)$

$J = (E_m - E_i)/(1+\nu)(1-2\nu)$

$K = J((1-\nu)B + \nu C + \nu F) - (1-\nu)G$

$L = J((1-\nu)C + \nu B + \nu F) - \nu G$

$M = J((1-\nu)D + \nu D + \nu E) - \nu G$

$N = J((1-\nu)D + \nu D + \nu E) - \nu G$

$P = J((1-\nu)B + \nu C + \nu F) - (1-\nu)G$

$Q = J((1-\nu)D + \nu D + \nu E) - \nu G$

$R = J((1-\nu)F + \nu B + \nu C) - \nu G$

$S = J((1-\nu)F + \nu C + \nu B) - \nu G$

$Y = J((1-\nu)B + 2\nu D) - (1-\nu)G$

$U = (PK - NL)/K$

$V = (QK - NM)/K$

$W = A'H(1+\nu)(N-K)/K$

$X = (SK - RL)/K)$

$Y = (TK - MR)/K$

$Z = A'H(1+\nu)(R-K)/K$

$X' = (ZU - XW)/(UY - XV)$

$Y' = (W - X'V)/U$

$Z' = (-Y'L - X'M - HA'(1+\nu))/K$

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APPENDIX D

FORMULA OF S_{ijkl} FOR A VARIETY OF INCLUSION

1. FORMULA OF $S_{ijkl}(\alpha=1.5-5)$

$$S_{1111} = S_{2222} = \frac{3}{8(1-\nu)}\frac{\alpha^2}{(\alpha^2-1)} + \frac{1}{4(1-\nu_0)}\left\{1 - 2\nu_0 - \frac{9}{4(\alpha^2-1)}\right\}g$$

$$S_{3333} = \frac{1}{2(1-\nu_0)}\left[\left\{1 - 2\nu_0 + \frac{(3\alpha^2-1)}{(\alpha^2-1)}\right\} - \left\{1 - 2\nu_0 + \frac{3\alpha^2}{(\alpha^2-1)}\right\}g\right]$$

$$S_{1122} = S_{2211} = \frac{1}{4(1-\nu_0)}\left\{\frac{\alpha^2}{2(\alpha^2-1)} - (1 - 2\nu_0) - \frac{3}{4(\alpha^2-1)}g\right\}$$

$$S_{1133} = S_{2233} = -\frac{1}{2(1-\nu_0)}\frac{\alpha^2}{(\alpha^2-1)} + \frac{1}{4(1-\nu_0)}\left\{\frac{3\alpha^2}{(\alpha^2-1)}\right.$$
$$\left. - (1 - 2\nu_0)\right\}g$$

$$S_{3311} = S_{3322} = -\frac{1}{2(1-\nu_0)}\left\{1 - 2\nu_0 + \frac{1}{(\alpha^2-1)}\right\}$$
$$+ \frac{1}{2(1-\nu_0)}\left\{1 - 2\nu_0 + \frac{3}{2(\alpha^2-1)}\right\}g$$

where ν_0 is Poisson's ratio of a matrix, α is aspect ratio of a fiber $(=l/d)$, and g is given by

$$g = \frac{\alpha}{(\alpha^2-1)^{3/2}}\left\{\alpha(\alpha^2-1)^{1/2} - \cosh^{-1}\alpha\right\}$$

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2. FORMULA OF $S_{ijkl}(\alpha=1)$

$$S_{1111} = \frac{7 - 5\nu}{15(1 - \nu)}$$

$$S_{1122} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{1133} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{2211} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{2222} = \frac{7 - 5\nu}{15(1 - \nu)}$$

$$S_{2233} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{3311} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{3322} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{3333} = \frac{7 - 5\nu}{15(1 - \nu)}$$

3. FORMULA OF $S_{ijkl}(\alpha=\infty)$

$$S_{1111} = \frac{5 - 4\nu}{8(1 - \nu)}$$

$$S_{1122} = \frac{4\nu - 1}{8(1 - \nu)}$$

$$S_{1133} = \frac{\nu}{2(1 - \nu)}$$

$$S_{2211} = \frac{4\nu - 1}{8(1 - \nu)}$$

$$S_{2222} = \frac{5 - 4\nu}{8(1 - \nu)}$$

$$S_{2233} = \frac{\nu}{2(1 - \nu)}$$

$$S_{3311} = 0$$

$$S_{3322} = 0$$

$$S_{3333} = 0$$

APPENDIX E

PROGRAM FOR CREEP DEFORMATION

PROGRAM(MATLAB)

```
ep0=0.001;
beta=0.0465;
k=7.2e-7;
for tt=1:91;
kv(tt)=tt;
t=1.0*10^(((tt-1)/30.));
eep(tt)=ep0*(1.+beta*t^(1/3))*exp(k*t);
end;
plot(kv,eep)
semilogx;
xlabel('Time t'),ylabel('Strain eps')
```

LIST OF REFERENCES

1. CHU–WHA LEE, "Thermal residual stress in Al/SiC Metal Matrix Composites and the influence of their viscoelstic and plastic relaxation on dimensional stability". RESEARCH PROPOSAL

2. J. D. ESHELBY, "The elastic field outside an ellipsoidal inclusion". "The determination of the elastic field of an ellipsoidal inclusion, and related problems". DEPARTMENT OF PHYSICAL METALLURGY, UNIVERSITY OF BIRMINGHAM. PROC. R. SOC. A242. 1957

3. K. WAKASHIMA, M. OTSUK and S. UMEKAWA "Thermal expansions of heterogeneous solids containing aligned ellipsoidal inclusions". J. COMPOSITE MATERIALS, VOL.8. 1974

4. R. J. ARSENAULT and M. TAYA, "Thermal residual stress in Metal Matrix Composite". ACTA METAL VOL.35 No.3 1987.

5. LEIF LARSSON and VOLVO, SWEDEN "Thermal stresses in MMCs" CSDL MMC PROGRAM CONSOLIDATION STRESSES

6. METAL HANDBOOK. VOL.8

7. M. TAYA and T. MURA, "On stiffness and strength of an aligned short–fiber reinforced composite containing fiber–end cracks under uniaxial applied stress". JOURNAL OF APPLIED MECHANICS, VOL.48, JUNE 1981

8. P. J. FRITZ and R. A. QUEENEY "Visco elastic and plastic relaxation of residual stresses around macroparticle reinforcements in metal matrices". COMPOSITES. VOL.14, OCT.1983

9. T. MORI and K. TANAKA, "Average stress in matrix and Average elastic ebergy of materials with misfitting inclusions". ACTA METALLURGICA, VOL.21, MAY 1973

10. Y. FLOM and R. J. ARSENAULT, "Deformation of SiC/Al composites". JOURNAL OF METALS, JULY 1986

11. TOSHIO MURA and M. TAYA, "Residual stresses in and around a short fiber in MMCs due to temperature change". RECENT ADVANCES IN COMPOSITES IN THE U.S and JAPAN, 1985

12. L M. BROWN AND D. R. CLARKE, "The work hardening of fibrous composites with particular preference to the COPPER–TUNGSTEN system". ACTA METALLURGICA. VOL.25. 1977

13. MARY VOGELSANG, R. J. ARSENAULT, AND R. M. FISHER "An IN SITU HVEM study of dislocation generation at Al/SiC interface in MMCs". METALLURGICAL TRANSACTIONS A. VOL. 17A 1986

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